Ameen Ismail
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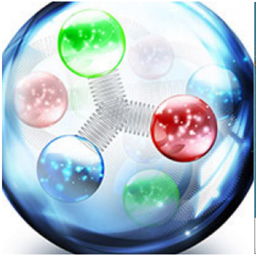
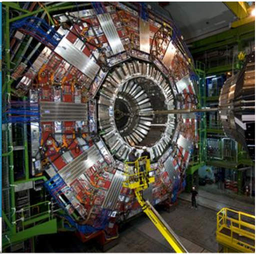
21 April 2024





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(towards QCD energy correlators from holography)

arXiv:2403.12123
with C. Csáki
CoSy HEP

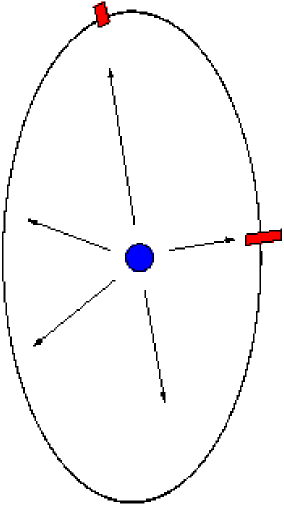
21 April 2024

Energy correlators

Correlation functions of energy flow operators $\langle \mathcal{E}(\vec{n}) \rangle$, $\langle \mathcal{E}(\vec{n})\mathcal{E}(\vec{n}') \rangle$

$\mathcal{E}(\vec{n})$ measures energy deposited in calorimeter in $\vec{n}$-direction

Considered in CFT context by Hofman + Maldacena '08

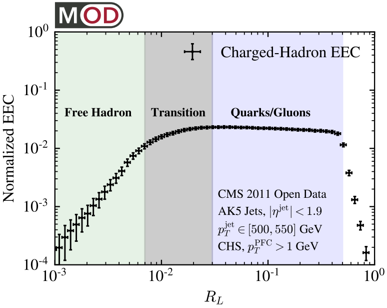


Energy correlators and confinement

Two-point function visualizes confinement transition

The big idea:

Can we reproduce this in a holographic model?



Komiske et al. 2201.07800

Calculating energy correlators

Correlation functions of energy flow operators

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt T_{0i}(t, x^i = rn^i)n^i$$

(take limit holding $t - r$ constant, sending $t + r \to \infty$)

In lightcone coordinates $x^\pm = t \pm x^3$, $x^\perp = x^{1,2}$:

$$\mathcal{E}(\vec{n}) = \lim_{x^+\to\infty} \frac{(x^+)^2}{4} \int_{-\infty}^\infty dx^- T_{--}(x^+, x^-, x^\perp)$$

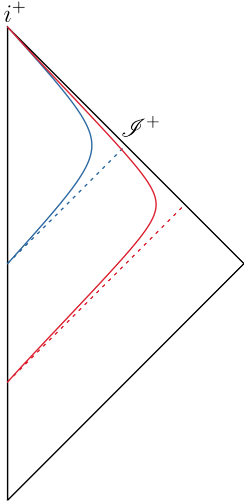
i.e. integral over future null infinity

A caveat about limits

Be **careful** about the order of limits!

- $x^- \to \pm\infty$ (integrate over time)
- $x^+ \to \infty$ (calorimeter to boundary)

Only important for gapped theories



The inversion

Avoid large-r limit with a conformal transformation:

$$x^+ \to -1/x^+, x^- \to x^- - \left|x^\perp\right|^2/x^+, x^\perp \to x^\perp/x^+$$

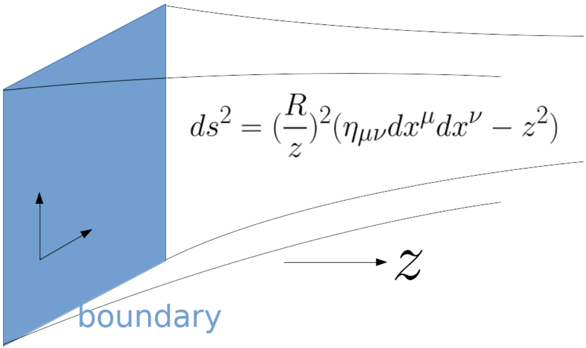
mapping $x^+ = \infty$ to $x^+ = 0$, $ds^2 \to ds^2/(x^+)^2$

Energy flow calculated as

$$\mathcal{E}(\vec{n}) = \left(1 + \left|x^\perp\right|^2\right)^3 \int_{-\infty}^{\infty} dx^- \, T_{--}(x^+ = 0, x^-, x^\perp)$$

with mapping to celestial sphere: $x^1 + ix^2 = e^{i\phi} \tan\theta/2$

Holographic calculation setup



We want a generating functional for correlators

Inserting an energy flow operator

Perturb

Belin et al. 2011.13862

$$\delta S_{\mathrm{CFT}} = \epsilon \int dx^- T_{--}(x^+ = 0, x^-, x^\perp = y^\perp)$$
$$= \epsilon \int d^4x T_{--} \delta(x^+) \delta^2(x^\perp - y^\perp)$$

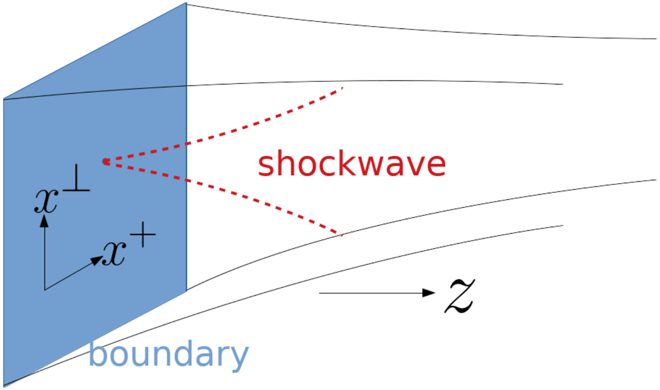
Inserts $e^{\epsilon \mathcal{E}}$ in path integral

Dual picture: shockwave,

$$\delta ds^2 = \frac{\epsilon}{z^2} \delta(x^+) f(x^\perp - y^\perp, z) \left(dx^+\right)^2$$

(boundary condition: $f(x^\perp, z = 0) = \delta^2(x^\perp)$)

Shockwave picture



Shockwaves in AdS

Field equations are linear: $\left(3/z\partial_z - \partial_z^2 - \partial_\perp^2\right) f = 0$

Shockwave

$$f(x^\perp, z) = \frac{z^4}{\left(z^2 + |x^\perp|^2\right)^3}$$

Shockwaves are linear $\Rightarrow$ superpose them: $ds^2 = ds^2_{\mathrm{AdS}} + \frac{\delta(x^+)}{z^2}\left[\epsilon_1 f(x^\perp - y_1^\perp, z) + \epsilon_2 f(x^\perp - y_2^\perp, z)\right]\left(dx^+\right)^2$

Exact solution of field equations

Scalar source

Scalar source, momentum $q^\mu = (q, \vec{0})$

$\Rightarrow$ bulk scalar ϕ with BC $\phi(z=0) = e^{iqt}$

ϕ gets a "kick" at the shockwave:

$$\lim_{\delta \to 0} \partial_- \phi(x^+ = \delta, x^-, x^\perp, z) = e^{-\epsilon f(x^\perp, z)\partial_-} \partial_- \phi(x^+ = -\delta, x^-, x^\perp, z)$$

(away from shockwave, usual AdS evolution)

Holographic correlators

One point: $\langle e^{\epsilon\mathcal{E}(y^\perp)}\rangle \sim$
$$\int \frac{dz}{z^3} d^2x^\perp dx^- \phi^* \exp\left[-\epsilon\left(1+(y^\perp)^2\right)^3 f(x^\perp - y^\perp, z)\partial_-\right]\partial_-\phi\Big|_{x^+=0}$$

Expanding in ϵ, find $\langle\mathcal{E}\rangle \sim 1$ (constant)

Higher-point functions from inserting more shockwaves, e.g. $\langle\mathcal{E}\mathcal{E}\rangle \sim 1$

Upshot

To find energy correlators, we need shockwave geometry

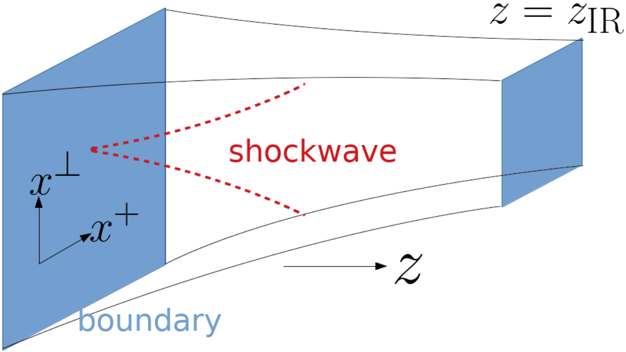
Constant correlators agree with strongly-coupled CFT expectation

Next: cut off with IR brane to model confinement

Cutting off AdS

Simplest "hard-wall" confinement model: introduce IR brane at $z = z_{\rm IR}$

Essentially RSI model with UV brane sent to AdS boundary



Shockwaves with a brane

Shockwave EOM unchanged, $(3/z\partial_z - \partial_z^2 - \partial_1^2 - \partial_2^2)f = 0$

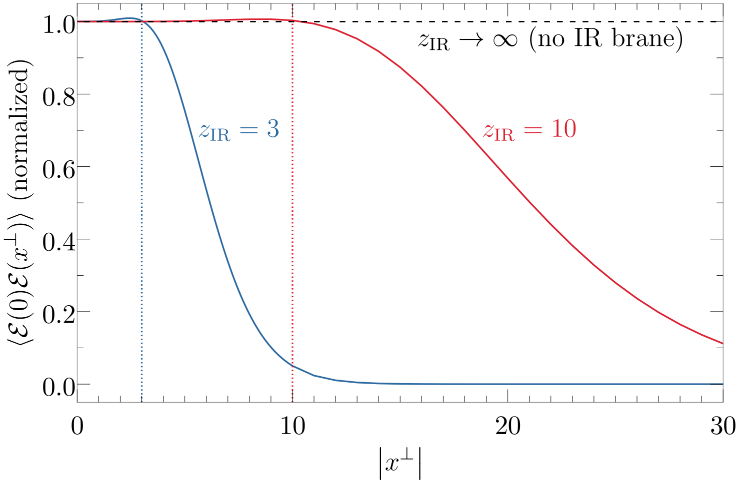
Brane modifies boundary condition to $\partial_z f(x^\perp, z)\Big|_{z=z_{\rm IR}} = 0$

Solution

$$f(x^\perp, z) = \frac{1}{8}\int_0^\infty dk\, J_0(kr)k^3 z^2 \left[K_2(kz) + \frac{K_1(kz_{\rm IR})}{I_1(kz_{\rm IR})} I_2(kz)\right]$$

(where $r = |x^\perp|$)

Results



1.0
0.8
0.6
0.4
0.2
0.0
$\langle \mathcal{E}(0)\mathcal{E}(x^\perp)\rangle$ (normalized)
0
10
20
30
$|x^\perp|$
$z_{\text{IR}} \to \infty$ (no IR brane)
$z_{\text{IR}} = 3$
$z_{\text{IR}} = 10$

Some comments

Scales $\ll z_{\mathrm{IR}}$: constant correlator, like strongly-coupled CFT

Scales $\gg z_{\mathrm{IR}}$: exponential decay

Manifest transition between confined and deconfined regimes

Outlook

Comparison with QCD correlators:

- constant correlator vs. asymptotic freedom
- exponential vs. power-law decay
- mapping back to location on celestial sphere?

Our methods should work for more realistic AdS/QCD backgrounds!

Thank you!

Photos: Jefferson Lab / Wikimedia Commons / CERN

more info:

arxiv.org/abs/2403.12123

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Wavefunction at the shock

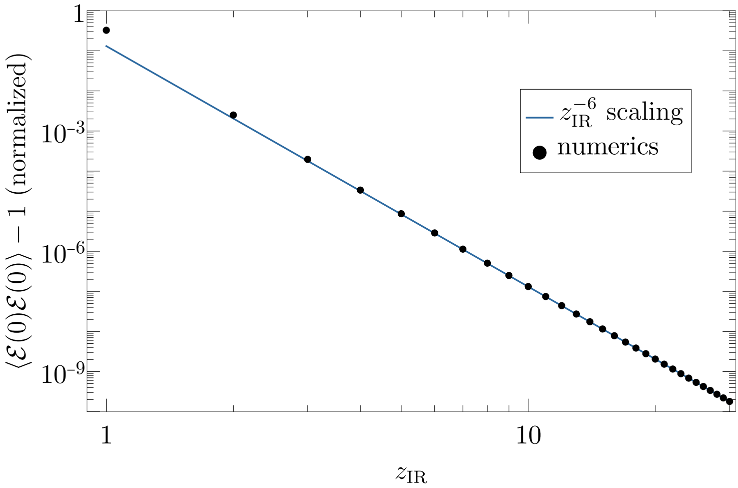
EOM:

$$\partial_- \partial_+ \phi + \epsilon \delta(x^+) f(x^\perp, z) \partial_-^2 \phi + \text{terms regular at shockwave} = 0,$$

Discontinuity:

$$\lim_{\delta \to 0} \partial_- \phi(x^+ = \delta, x^-, x^\perp, z) = e^{-\epsilon f(x^\perp, z) \partial_-} \partial_- \phi(x^+ = -\delta, x^-, x^\perp, z)$$

More figures 1



More figures 2