PHYS2217 discussion

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13 April 2020

1 Induced electric field

Consider an infinitely long wire carrying a current $I(t) = I\cos\omega t$. Assume ω is sufficiently small that you can use the quasistatic approximation. (How small is "sufficiently" small? Think about it!) This induces an electric field everywhere. Why? Compute the induced electric field $\vec{\mathbf{E}}(s)$ at a distance s from the wire.

Hints: recall Faraday's law is $\int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\ell} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$. What surface should you apply Faraday's law to? Well, Faraday's law is pretty similar in structure to Ampère's law, so perhaps the Amperian loop you consider to compute the magnetic field from a steady line current would be a good place to start...

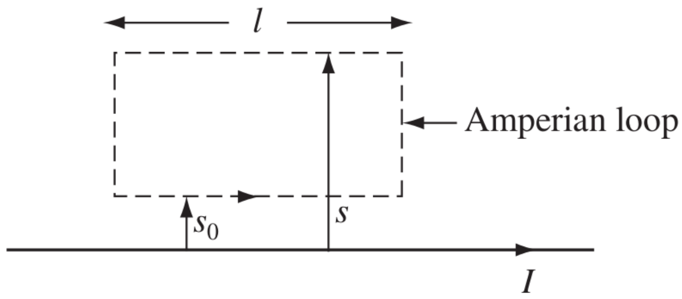

Figure 1: Diagram for problem, cropped from Griffiths §7.2.2.

Solution: induced electric field

Using the quasistatic approximation, the magnetic field produced by the current is simply

$$\vec{\mathbf{B}} = \frac{\mu_0 I(t)}{2\pi s}\hat{\phi} = \frac{\mu_0 I}{2\pi s}\cos\omega t\hat{\phi}. \tag{1}$$

We now apply Faraday's law to the rectangular loop shown in the diagram. First we must compute the magnetic flux through the loop. This is just

$$\Phi = \int \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\mathbf{a}} = \int_0^L \mathrm{d}z \int_a^b \mathrm{d}r\, \frac{\mu_0 I(t)}{2\pi r} = \frac{\mu_0 L I(t)}{2\pi}\log b/a. \tag{2}$$

From this it follows that

$$-\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\mu_0 L I}{2\pi}\omega\sin(\omega t)\log b/a \tag{3}$$

Now consider the other side of Faraday's law, $\int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\ell}$. We know that $\vec{\mathbf{E}}$ can only point in the $\hat{\mathbf{z}}$ direction. Recall that in differential form Faraday's law is $\boldsymbol{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$. Since $\vec{\mathbf{B}}$ points in the $\hat{\phi}$ direction, $\vec{\mathbf{E}}$ must then point in the $\hat{\mathbf{z}}$ direction. Therefore, only the sides of the rectangle parallel to the wire contribute to the line integral. We find

$$\int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\ell} = L(E(a) - E(b)). \tag{4}$$

Putting this all together, we have

$$L(E(a) - E(b)) = \frac{\mu_0 L I}{2\pi}\omega\sin(\omega t)(\log b - \log a). \tag{5}$$

From this we can read off the solution:

$$E(s) = -\frac{\mu_0 \omega I}{2\pi}\sin(\omega t)\log(s) + C \tag{6}$$

where C is a constant, independent of s.

Some comments are in order. First, this begs the question: what is C? How can we determine this constant? Second, notice that our expression for $E(s)$ blows up at large s! What's going on here? The quasistatic approximation is no longer valid at large s. Electromagnetic fields propagate at the speed of light. The only time scale we have in our problem is ω^{-1}. This implies a distance scale of c/ω. At this distance, the time it takes for changes in current $I(t)$ to induce changes in the electric field is comparable to the time scale on which the current itself is varying.